

Supersymmetry and finite-temperature strings ^{*}

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Abstract

We describe finite temperature $N = 4$ superstrings in $D = 5$ by an effective four-dimensional supergravity of the thermal winding modes that can become tachyonic and trigger the instabilities at the Hagedorn temperature. Using a domain-wall ansatz, exact solutions to special BPS-type first order equations are found. They preserve half of the supersymmetries, contrary to the standard perturbative superstring at finite temperature that breaks all supersymmetries. Our solutions show no indication of any tachyonic instability and provide evidence for a new BPS phase of finite temperature superstrings that is stable for all temperatures. This would have important consequences for a stringy description of the early universe.

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1 Introduction

A d -dimensional field theory at finite temperature $T = \frac{1}{\beta}$ is formulated as a theory with a Euclidean time taking values on a circle of circumference β . There can then be no time dependence any more, in agreement with temperature being only well-defined in a time-independent equilibrium. Bosonic fields must be periodic around the circle while fermionic ones must be antiperiodic [1]. This leads to different Fourier modes, i.e. to different thermal masses for bosons and fermions. Thus, if one starts with a supersymmetric theory, introducing finite temperature breaks the supersymmetry.

In (closed) superstring theory things are similar but more subtle. Whenever a dimension is compactified, the closed string can wind an arbitrary number of times around this circle. At zero temperature, when an ordinary spatial dimension is compactified the spectrum of the superstring is modified by the appearance of corresponding momentum and winding states in a modular invariant way. These give a positive contribution to the masses of the corresponding states. At finite temperature however, we must recover the different boundary conditions for fermions and bosons around the thermal circle. Modular invariance then fixes all signs for the contributions of the different spin structures to the partition function [2]. This results in a GSO projection different from the usual one at zero temperature. In particular, at zero temperature the state with no oscillator excitations (in the RNS formalism) would be a tachyon and is eliminated by the GSO projection. At finite T however, such a state, with unit winding number, is kept by the modified GSO projection and its mass squared is $-8\alpha' + \frac{\beta^2}{\pi^2}$. For large β (small T) this is positive, but it turns negative at $T = T_H \equiv \frac{1}{\pi\sqrt{8\alpha'}}$, called the Hagedorn temperature. Thus at high temperature certain winding modes turn tachyonic, signaling an instability and a transition to a new phase where the corresponding field has acquired a vacuum expectation value.

Usually the existence of this instability for $T \geq T_H$ is interpreted as due to the non-convergence of the thermal partition function $Z = \text{Tre}^{-\beta H}$ because the number of string states contributing to the n^{th} level grows like $e^{c\sqrt{n}}$. This reason looks very different from the above argument but one should realize that in string theory modular invariance relates this exponential growth at high energies to the behavior of the low-lying momentum and winding modes.

Since one can view the thermal instability as solely due to certain winding modes becoming tachyonic, it should be possible to get an accurate description of the physics near the critical temperature by studying an effective theory of these modes (fields). In refs. [3, 4] this effective theory was found for all known five-dimensional $N = 4$ superstring theories at finite temperature, which effectively are four-dimensional. A relevant subset of potentially tachyonic winding modes was isolated from the non-perturbative BPS mass formula, and an effective supergravity theory for these modes, coupled to the usual dilaton and temperature modulus, was constructed. This theory is briefly reviewed in section 2. The supergravity potential clearly exhibits domains where tachyonic modes are present.

Rather than looking at perturbation theory of this supergravity (which one could do directly in the full string theory), we try to find exact solutions [5, 6]. To do so, we make a domain-wall ansatz for the metric and use a certain technique (more or less well-known from recent studies of RG-flows in the AdS/CFT correspondence) to reduce the second order equations to first order ones. This is reminiscent of BPS conditions and, indeed, we have proven [5] that all solutions of the first order equations preserve half of the supersymmetries, i.e. they are really BPS. This will be described in section 3.

The full set of first order equations is still too complicated to be solved exactly, but certain consistent truncations lead to various type II or heterotic theories. The solutions in each of these cases have been described in great detail in ref. [5] and also in ref. [6]. Here, in section 4, we describe only their main properties. A striking feature is that almost all of these solutions interpolate between low and high temperature, typically including regions of strong coupling. They exhibit no sign of instability or phase transition whatsoever. This apparent stability fits well with their BPS property.

The usual instability of strings as the Hagedorn temperature T_H is reached is obtained from a perturbative analysis where supersymmetry is completely broken by the finite temperature. Instead, we have found solutions which preserve half of the supersymmetries. We are tempted to conclude that they correspond to a new, more stable phase of superstrings (or at least of the effective supergravity) which does not undergo any phase transition as the temperature is raised. This may have important consequences in a stringy description of the early universe.

2 Review of the Effective Thermal Supergravity

One starts with the six-dimensional $N = 4$ closed superstring theories (heterotic on T^4 or type II on $K3$), compactifies one dimension on a circle of radius R_6 and the Wick-rotated Euclidean time on the thermal circle of radius $R = (2\pi T)^{-1}$. As discussed above, the GSO projection has to be changed, resulting in a modified thermal mass formula. It is enough to restrict the attention on those modes that become tachyonic first as the temperature is raised. These are certain dyonic modes with winding numbers ± 1 . There are three such \pm modes. Which pair of them turns tachyonic first depends on the values of the moduli $s = \sqrt{2}/g_{\text{het}}^2$, $t = \sqrt{2}RR_6/\alpha'_{\text{het}}$ and $u = \sqrt{2}R/R_6$ (the four-dimensional gravitational coupling constant is normalized as $\kappa = \sqrt{2}$ and $\alpha'_{\text{het}} = 4s$), which determine whether one is in the heterotic, type IIA or type IIB sector. Altogether, one has to find an effective supergravity for the 3×2 dyonic modes and the 3 moduli s, t, u . The restriction to these modes only truncates the theory from $N = 4$ to an $N = 1$. This provides enough constraints to determine the (bosonic part of the) effective theory (see ref. [4] for details and ref. [5] for a shorter review). It is of the general form

$$S = \int d^4x \sqrt{G} \left(\frac{1}{4}R - \frac{1}{2}K_{IJ}\partial^\mu\Phi^I\partial_\mu\bar{\Phi}^J - V(\Phi, \bar{\Phi}) \right), \quad (2.1)$$

where Φ^I , $I = 1, 2, \dots, 9$ denotes the bosonic parts of the chiral superfields and $K_{I\bar{J}}$ is the Kähler metric derived from a Kähler potential K . The potential V is given by $V = \frac{1}{4} e^K \left(K^{I\bar{J}} W_{;I} \bar{W}_{;\bar{J}} - 3W\bar{W} \right)$ where $W_{;I} = \frac{\partial W}{\partial \Phi^I} + \frac{\partial K}{\partial \Phi^I} W$ and W is the superpotential. The potential and the superpotential were given in ref. [4].

The analysis of the thermal mass spectrum in ref. [4] reveals that the directions in which tachyonic instabilities could occur are the real directions. It is thus enough to restrict oneself to this case, where we have a total of 9 real fields. Writing $s = e^{-2\phi_1}$, $t = e^{-2\phi_2}$, $u = e^{-2\phi_3}$ and denoting the real parts of the truncated dyonic modes as $z_a^\pm = \text{Re } Z_a^\pm$ one finds ($x_\pm^2 \equiv \sum_{b=1}^3 (z_b^\pm)^2$)

$$S = \int d^4x \sqrt{G} \left(\frac{1}{4} R - \frac{1}{2} \sum_{i=1}^3 (\partial_\mu \phi_i)^2 - \sum_{a=1}^3 \frac{(\partial_\mu z_a^+)^2}{(1-x_+^2)^2} - \sum_{a=1}^3 \frac{(\partial_\mu z_a^-)^2}{(1-x_-^2)^2} - V \right). \quad (2.2)$$

All we need to know about the potential V as a function of the real fields is that it can be expressed as

$$V = \frac{1}{4} \sum_{i=1}^3 \left(\frac{\partial \widetilde{W}}{\partial \phi_i} \right)^2 + \frac{1}{8} \sum_{a=1}^3 (1-x_+^2)^2 \left(\frac{\partial \widetilde{W}}{\partial z_a^+} \right)^2 + \frac{1}{8} \sum_{a=1}^3 (1-x_-^2)^2 \left(\frac{\partial \widetilde{W}}{\partial z_a^-} \right)^2 - \frac{3}{4} \widetilde{W}^2, \quad (2.3)$$

where the real superpotential \widetilde{W} is given in terms of the superpotential W as $\widetilde{W} = e^{K/2} W|_{\text{real directions}}$. In the present case

$$\widetilde{W} = \frac{1}{2} e^{\phi_1 + \phi_2 + \phi_3} - 2e^{\phi_1} \sinh(\phi_2 + \phi_3) H_1^+ H_1^- + e^{-\phi_1} \left(e^{\phi_2 - \phi_3} H_2^+ H_2^- + e^{\phi_3 - \phi_2} H_3^+ H_3^- \right) \quad (2.4)$$

where $H_a^\pm \equiv \frac{z_a^\pm}{1-x_\pm^2}$. The form of the theory (2.2)-(2.4) which is obtained when the $+$ and $-$ winding modes are restricted to behave in exactly the same way, i.e. $z_a^+ = z_a^-$, was given in ref. [5], whereas the more general form that is presented here can be found in ref. [6]. In the rest of this contribution we restrict our discussion for simplicity to the case of $z_a^+ = z_a^- \equiv z_a$ following ref. [5] while for its generalization to the case with $z_a^+ \neq z_a^-$ we refer the reader to ref. [6].

3 First Order BPS Equations and Supersymmetry

We make the domain-wall ansatz for the four-dimensional (Euclidean) metric:

$$ds^2 = dr^2 + e^{2A(r)} (dx_1^2 + dx_2^2 + dx_3^2) \quad (3.5)$$

and suppose that all scalar fields only depend on the single coordinate r . It is then straightforward to verify that the second order equations of motion for the metric and the scalar fields φ^I appearing in eq.(2.2) ($\varphi^I = \phi_i$ or z_a) are satisfied if the $\varphi^I(r)$ and the conformal factor $A(r)$ are solutions of the first order equations

$$\frac{d\varphi^I}{dr} = \pm \frac{1}{\sqrt{2}} K^{I\bar{J}} \frac{\partial \widetilde{W}}{\partial \varphi^{\bar{J}}}, \quad \frac{dA}{dr} = \mp \frac{1}{\sqrt{2}} \widetilde{W}. \quad (3.6)$$

Such first order gradient equations exist quite generally for the domain-wall ansatz if the potential V can be expressed in terms of a real superpotential \widetilde{W} as in (2.3). Given the relation between V, \widetilde{W} and W , this only depends on the real directions and a certain reality property of the Kähler potential, which are realized in a broad class of theories.

One can now study the supersymmetry transformations of the fermions to check whether a given bosonic solution φ breaks or preserves the supersymmetries. For example, one has for the left-handed fermionic component χ_L^I associated with φ^I :

$$\delta\chi_L^I = \frac{1}{\sqrt{2}}\gamma^\mu(\partial_\mu\varphi^I)\epsilon_R - \frac{1}{2}\epsilon_L e^{K/2}K^{I\bar{J}}W_{;\bar{J}}. \quad (3.7)$$

Since only $\partial_r\varphi^I$ is non-vanishing and $e^{K/2}K^{I\bar{J}}W_{;\bar{J}} = K^{I\bar{J}}\frac{\partial\widetilde{W}}{\partial\varphi^{\bar{J}}} = \pm\sqrt{2}\partial_r\varphi^I$, we have

$$\delta\chi_L^I = \frac{1}{\sqrt{2}}(\gamma^r\epsilon_R \mp \epsilon_L)\partial_r\varphi^I = \frac{1}{\sqrt{2}}\partial_r\varphi^I P_L(\gamma^r \mp 1)\epsilon. \quad (3.8)$$

Then, since $\frac{1}{2}(1 \mp \gamma^r)$ is a projector, half of the components of ϵ give $\delta\chi_L^I = 0$, i.e. the solutions φ^I preserve half of the supersymmetries: they are BPS. Of course, one also has to check that the supersymmetry variation of the gravitino vanishes. This is indeed the case provided $\epsilon(r) = e^{A(r)/2}\epsilon_0$, see ref. [5] for more details.

The explicit set of six non-linear first-order ordinary differential equations for ϕ_1, ϕ_2, ϕ_3 and z_1, z_2, z_3 is easily obtained from (3.6) using the prepotential (2.4) and the Kähler metric following from (2.2). We will not write down the eqs. in detail, as they can be found in ref. [5]. We also note that the considerations of this section apply equally well to a general four-dimensional $N = 1$ supergravity theory coupled to any number of chiral multiplets.

4 Truncations to Specific Sectors

Type II: We further truncate the system to be able to find exact solutions: Letting $z_1 = z_2 = 0$, $z_3 \equiv \tanh\left(\frac{\omega}{2}\right)$, $\phi_1 = \phi_2 \equiv \phi/\sqrt{2}$ and $\phi_3 \equiv \chi$ leads to type IIB, while letting $z_1 = z_3 = 0$, $z_2 \equiv \tanh\left(\frac{\omega}{2}\right)$, $\phi_1 = \phi_3 \equiv \phi/\sqrt{2}$ and $\phi_2 \equiv \chi$ leads to type IIA. In both cases one obtains the same \widetilde{W} and the same equations when expressed in terms of ϕ, χ and ω . They can be solved exactly as (c is a constant of integration)

$$e^{-2\sqrt{2}\phi} = 2 \cosh^2 \omega (\ln \coth^2 \omega + c) - 2, \quad e^{2\chi} = \cosh^2 \omega e^{\sqrt{2}\phi}, \quad (4.9)$$

with $e^{-2\sqrt{2}\phi} \sim T^{-2}$ and $e^\chi \sim g_{\text{II}}$ being the string coupling. The metric is

$$ds^2 = \frac{8e^{\sqrt{2}\phi}}{\sinh^2 \omega \cosh^4 \omega} d\omega^2 + \frac{e^{-\sqrt{2}\phi}}{\cosh^2 \omega} (dx_1^2 + dx_2^2 + dx_3^2). \quad (4.10)$$

with r and ω related by $dr = -\frac{2\sqrt{2}e^{\phi/\sqrt{2}}}{\sinh \omega \cosh^2 \omega} d\omega$. One can see from the potential V that ω is tachyonic if $e^{-2\sqrt{2}\phi} < 4$ at $\omega = 0$. However, it is obvious from the explicit solution (4.9)

that, independently of the value of c , one has $T^{-2} \sim e^{-2\sqrt{2}\phi} \sim 2 \ln \omega^{-2} \rightarrow \infty$ as $\omega \rightarrow 0$. Thus the winding mode ω does not become tachyonic. Also, in all cases, the solution interpolates from weak to strong coupling ($e^{2\chi} \rightarrow \infty$).

We also note that a different type of truncation having $z_1 = 0$, $z_2 = \pm z_3 \equiv \frac{1}{\sqrt{2}} \tanh \frac{\omega}{2}$, $\phi_2 = \phi_3 \equiv \phi/\sqrt{2}$, $\phi_1 \equiv \chi$, leads to a self-dual type II or hybrid type II theory. This was presented in great detail in refs. [5, 6] and will not be repeated here.

Heterotic sector: Finally, the heterotic sector is obtained by the truncation $z_2 = z_3 = 0$, $z_1 \equiv \tanh \frac{\omega}{2}$, $\phi_2 = \phi_3 \equiv \phi/\sqrt{2}$ and $\phi_1 \equiv \chi$. Unfortunately, we have not been able to solve the corresponding three equations in closed form. In ref. [5] a detailed study of various asymptotic regimes, as well as in the vicinity of a critical point of the differential equations has been performed, yielding a coherent global picture of how the solutions behave. There is one weakly coupled region $\phi \rightarrow -\infty$, $\omega \rightarrow 0$ and three strongly coupled ones; region 1: $\phi \rightarrow \infty$ and $\omega \rightarrow 0$, region 2: $\phi \rightarrow \infty$ and $\omega \rightarrow \infty$, region 3: $\phi \rightarrow -\infty$ and $\omega \rightarrow \infty$, as well as a special critical point for $\phi = 0$ and $\omega = \ln(\sqrt{2} + 1)$. There are solutions going from the special critical point to any of the 3 strong coupling or the weak coupling regions, as well as solutions extending from weak coupling to regions 1 or 3 and solutions from region 2 to 1 or 3. We refer the reader to ref. [5] for an in depth discussion of these solutions.

One expects that certain physical criteria discriminate between the various solutions, in particular between different ranges of the integration constants. For example, one can look at the metric and study the propagation of a test particle (wave) in this supergravity background. In general the metrics exhibit certain singularities which may or may not be admissible. We have found [5] that they are admissible if the integration constant $c \leq 0$ for the type II case. For the heterotic case, solutions that contain region 3 are not, while all others are admissible. It is less clear which solutions are admissible when studying the string, rather than the particle propagation in these backgrounds. Another interesting question is the stability of solutions against small fluctuations, which is expected for supersymmetric solutions. This is currently under study.

5 Conclusions and speculations

We have described finite temperature strings in $D = 5$ by an effective four-dimensional supergravity. Using a domain-wall ansatz, exact solutions to special BPS-type first order equations have been found. They preserve half of the supersymmetries, contrary to the standard perturbative solution at finite temperature that breaks all supersymmetries. They show no indication of any tachyonic instability, since the domain of (ϕ, ω) or (T, ω) where this instability could occur is avoided by these solutions. If this behaviour extends beyond the effective supergravity to the full string theory, these solutions will describe a new BPS phase of finite temperature superstrings that is stable for all T .

An issue concerning our supersymmetric solutions is whether they suffer from Jeans

instabilities, typical in thermodynamical systems that contain gravity, as the example of hot flat space studied in ref. [7]. One could be tempted to conclude that such instabilities cannot be avoided in our solutions especially due to the fact that they do not support small volumes and according to general arguments they should collapse into black holes [2]. However, the counter argument is that the supersymmetry of our solutions will ensure their quantum stability and that no gravitational collapse will occur. We note at this point that the analysis of ref. [7] was done around hot flat space and supersymmetry was not even an issue. Our spaces have very different asymptotics and we think that the conclusions of ref. [7] are not directly applicable to our cases.

If we want to look for fingerprints of string theory in observational data, a good place to study will be the early universe, and whatever relics may be observed today. A stringy description of the early universe certainly should include finite temperature and actually high temperature effects. It is conceivable that the differences found between a stable BPS high temperature phase and a perturbative phase with broken supersymmetry may lead to important changes that eventually will be observed and tested. We leave these exciting questions for further study.

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